MPM 2D Review for Exam

Chapter 1: Systems of Equations

Major New Skills:

- Solving a linear system by graphing, substitution, eliminating by adding or subtracting
- Solving problems involving systems of linear equations based on
  - Break even situations
  - Mixtures
  - Speed, distance and time combinations
  - Situations involving word descriptions of mathematics operations

Sample Questions

1. Solve $\begin{cases} 2x+3y=10 \\ x+y=4 \end{cases}$ by graphing.

2. Solve $\begin{cases} \frac{3}{2}x-5y=8 \\ 2x+y=3 \end{cases}$ by substitution.

3. Solve $\begin{cases} 2x+3y=6 \\ 15x-21y=-44 \end{cases}$ by eliminating a variable by adding or subtracting.

4. Describe how you can decide whether a system of two linear equations has no solution, one solution or an infinite number of solutions, without actually solving the system.

5. A hockey player is offered two options for a contract: either a base salary of $50 000 and $1 000 per goal, or a base salary of $40 000 and $1500 per goal. How many goals must he score in order to make the same amount of money for each contract?

6. A Porche is driven 1.6 times as fast as a BMW. If the difference between their speeds is 45 km/h, how fast is the Porche driven?

7. A lab technician needs 5 L of a solution containing 8 % salt. Unfortunately, her supply solutions are 4 % salt and 11% salt. How many litres of each supply solution must she combine to get the 5 L of 8 % solution?

8. Jeff travels 2 h by car and 3 h by bus. The average speed of the bus is 20 km/k slower than that of the car, and he goes 40 km farther by bus. What was the average speed of the car, and how far was Jeff's entire trip?

Chapter 2: Coordinate Geometry

Major New Skills:

- Calculating length of line segment, coordinates of midpoints
- Circles centred on the origin: determining equation for, radius of, whether a given point is inside, outside or on
- Classifying shapes by using slopes and lengths
- Verifying properties by using slopes and lengths
- Determining equations of altitudes, medians and perpendicular bisectors
- Determining coordinates of centroids, orthocentres and circumcentres

Sample Questions

All of the questions are based on the triangle whose vertices are $A(3,5), B(-2,0), C(2,-4)$.

1. Calculate the exact length of AC.

2. Determine the coordinates of the midpoint of AB.

3. What is the equation of the circle centred on the origin that passes through C?

4. Is point A inside, outside or on the circle in question #3?

5. Determine the equation for
   (a) the altitude from B.
   (b) the median from A.
   (c) the perpendicular bisector of AC.

6. Determine the coordinates of the circumcentre of the triangle.

7. Determine whether ΔABC is right angled, by using two different methods.

8. Determine whether ΔABC is equilateral, isosceles or scalene.

9. Determine the coordinates of a point D such that $ACBD$ is a parallelogram.

Chapter 3: Analyzing and Applying Quadratic Models

Major New Skills:

- Determine whether data is quadratic by calculating second differences
- Determine the zeros, axis of symmetry, vertex and y-intercept of a parabola whose equation is given in factored form, and using these things to sketch a graph of the parabola
- Converting from standard form to factored form.
- Factoring $ax^2 + bx + c$
- Solving quadratic equations by factoring
- Solving problems involving zeros, optimal values and other points on parabolas

Sample Questions

1. Decide whether the following data is quadratic, and then complete the chart.

x	0	3	6	9	12
y	6	-4	-10	-12	

2. The zeros of a quadratic relation are 3 and -5, and the second differences are negative.
   (a) Will the optimal value be positive or negative? Explain.
   (b) If the y-intercept is -8, determine the coordinates of the vertex.
   (c) Make a sketch of the graph of the relation.

3. Expand and simplify:
   (a) $-3(x-5)(2x+7)$
   (b) $2(4x+3)^2$

4. Factor completely
   (a) $6x^2-x-12$
   (b) $3x^2+9x-30$
   (c) $27-12y^2$
   (d) $4b^2+20b+25$

5. Convert $y=0.2x^2-2x+3.2$ into factored form.

6. Solve:
   (a) $x^2+12x+35=0$
   (b) $3h^2-39h=-126$

7. The value, V, of the MegaCash company is modeled by $V=0.2t^2-2t+4.2$, where V is measured in millions of dollars and t is measured in years, with t=0 in the year 2000.
   (a) What was the value of the company in 2002?
   (b) In what year(s) did the value of the company equal 1 million dollars?
   (c) Between what years did the company have to borrow money in order to survive?

8. As owner of *The Wilted Bloom* floral company, Dave knows that he can sell 60 roses every day if he charges $1 per rose. He also knows that for every 10 cents he increases the price, he sells 2 fewer roses.
   (a) Create a quadratic relation to model his revenue.
   (b) Use your model to determine the price for a rose will maximize his revenue.

Chapter 4: Graphing Parabolas and Using Them as Models

Major New Skills

- Determine the vertex, axis of symmetry, direction of opening and y-intercept from the vertex form of a quadratic relation
- Determine the equation of a parabola, given the coordinates of the vertex and one other point on the parabola
- Describe how the base parabola $y = x^2$ can be transformed to a parabola whose equation is given in vertex form
- Determine the coordinates of the vertex of a parabola whose relation is given in standard form by:
  - Partial factoring
  - Completing the square
- Solving a quadratic equation by:
  - isolating the variable after completing the square ("inverting")
  - using the quadratic formula
- Using the discriminant to determine whether a quadratic equation has 0, 1 or 2 roots.
- Modeling parabolic shapes by imposing a coordinate system
- Modeling area problems using quadratic relations

Sample Questions

1. Give the coordinates of the vertex, the equation of the axis of symmetry, and the y-intercept of the parabola whose equation is $y = -3(x+5)^2 + 8$. Also, state whether the optimal value for this relation is a maximum or a minimum value of the relation.

2. A parabola has vertex (4, -2) and a zero when x=6. Determine the equation of the parabola, and make a sketch of the parabola.

3. Describe the graph of $y = -\frac{1}{2}(x+6)^2$ compared to the graph of $y = x^2$.

4. Determine the coordinates of the vertex of the following by using the method of partial factoring:
   (a) $y = -2x^2 + 8x + 9$
   (b) $y = \frac{1}{2}x^2 - 7x - 4$

5. Write the following in vertex form by completing the square:
   (a) $y = x^2 + 10x - 8$
   (b) $y = 2x^2 + 9x - 1$
   (c) $y = -3x^2 + 5x + 7$

6. Solve by using the quadratic formula. If the answers are not exact, round to 3 decimal places.
   (a) $x^2 - 8x - 20 = 0$
   (b) $3x + 2 - 8x^2 = 0$

7. Without solving the equation, determine the number of solutions to $2x^2 + 6x + 13 = 0$.

8. Solve $-3(x+2)^2+8=0$, rounding your answers to the nearest tenth.

9. A concrete bridge has an underside in the shape of a downward opening parabola. The bridge at the waterline is 40 m wide and its maximum height above the water is 12 m.
   (a) Draw a labeled diagram and use it to model the bridge.
   (b) Use your model to calculate, to the nearest cm,
      (i) how high the bridge is above the water at a point 2 m in from its base
      (ii) how wide the bridge would be at its base if the water level were to rise 2 m.

10. A sidewalk of uniform width is to be built around a rectangular 8 m X 10 m garden. If the area of the sidewalk is to be one half the area of the garden, how wide should the sidewalk be?

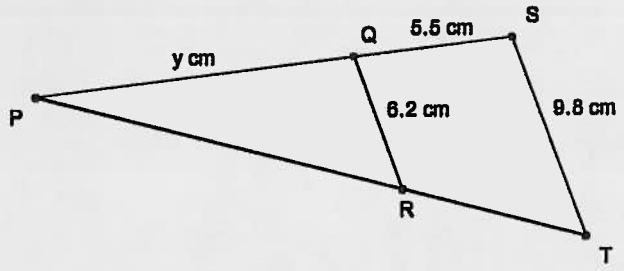
Chapters 5 and 6: Triangles

Major New Skills

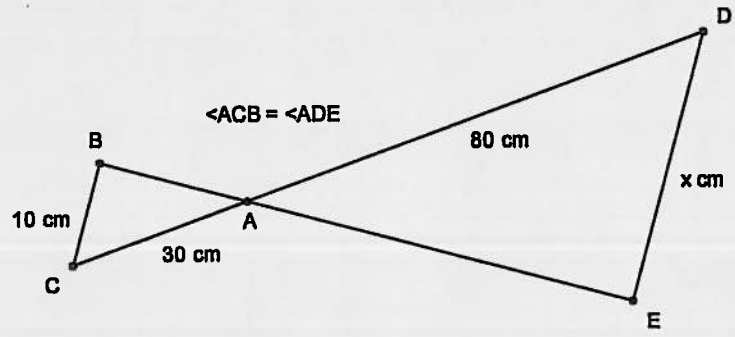
- Explaining whether a pair of triangles are congruent, similar, or neither
- Calculating the lengths of sides on pairs of similar triangles
- Determining the angle that a line segment makes with the horizontal
- Given 2 sides of a right angled triangle, determine the 3 primary trig ratios for either of the acute angles.
- Use trig ratios to calculate the length of missing sides, or the size of missing angles, for right angled triangles
- Using the sine and cosine laws to determine angles or sides in any triangle
- Solving word problems based on triangles

Sample Questions

1. State which triangles are similar, with reasons, and calculate the values of the unknowns.

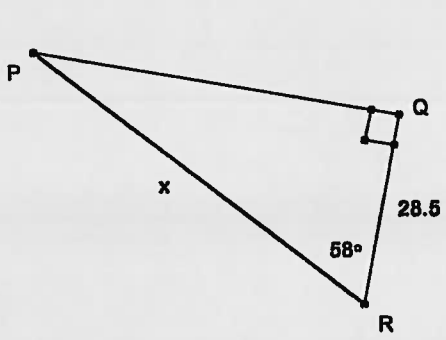


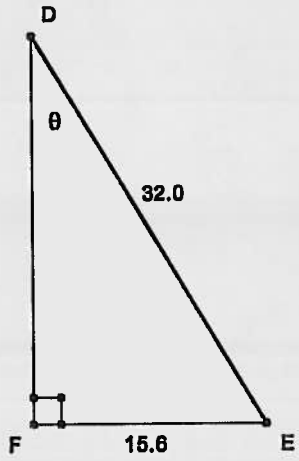
QR is parallel to ST

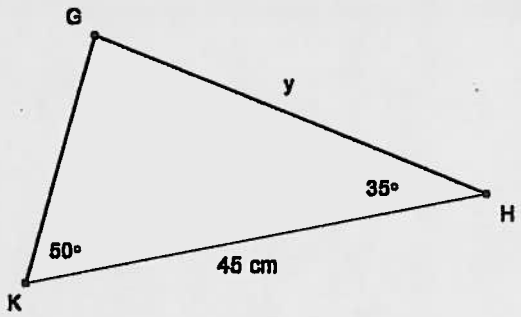


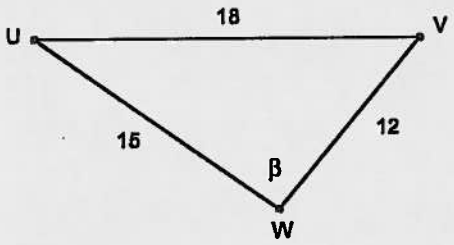
2. What is the acute angle between the lines $y = 3x + 5$ and $3x + 5y = 8$, to the nearest degree?

3. Solve for the unknowns in the following triangles.









4. Amanda places a mirror on the ground 7.5 m in front of the base of a flagpole. If she stands back 1.2 m from the mirror, she can see the reflection of the top of the pole in the mirror. If Amanda is 1.6 m tall, how tall is the flagpole?

5. From the edge of the roof of a building, the angle of depression of the base of a neighboring building is 28 °. If the two buildings are 50 m apart, how tall is the building from which the angle was measured?

6. $\Delta ABC \sim \Delta DEF$ with AB = 12.5 cm, BC = 20.8 cm, DF = 8.2 cm and EF = 5.2 cm. Calculate the length of AC. What is the scale factor for ΔDEF compared to ΔABC?

7. A helicopter is hovering above a spot between Ben and Vanessa, who are standing on level ground 600 m apart. The angles of elevation as measured by Ben and Vanessa are 35 ° and 42 ° respectively. How far is the helicopter from Ben?

8. A boat travels 45 km in the direction E 40 ° N, then turns and travels 25 km straight N. How far is the boat from its original starting point, and in what direction should it head to get back to there?

Answers:

Chapter 1

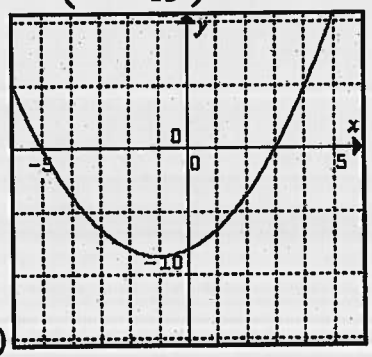
1. $x=2, y=2$
2. $x=2, y=-1$
3. $x=-\frac{2}{29}, y=\frac{178}{29}$
4. see page 67 of text
5. 20
6. 120 km/h
7. $\frac{15}{7}$L of 4%, $\frac{20}{7}$L of 11%
8. 100 km/h, 440 km

Chapter 2

1. $\sqrt{82}$
2. $\left(\frac{1}{2},\frac{5}{2}\right)$
3. $x^2+y^2=20$
4. outside
5. (a) $y=-\frac{1}{9}x-\frac{2}{9}$
   (b) $y=\frac{7}{3}x-2$
   (c) $y=-\frac{1}{9}x+\frac{7}{9}$
6. $\left(\frac{5}{2},\frac{1}{2}\right)$
7. right angled at B
8. $(7,1)$

Chapter 3

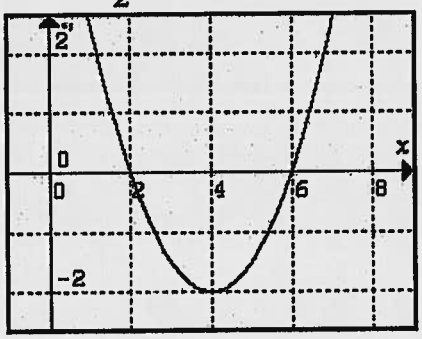
1. is quadratic, 10
2. (a) negative, opens up
   (b) $\left(-1,-\frac{128}{15}\right)$
   (c)



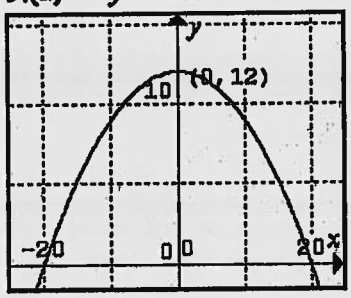
3. (a) $-6x^2+9x+105$
   (b) $32x^2+48x+18$
4. (a) $(2x-3)(3x+4)$
   (b) $3(x+5)(x-2)$
   (c) $3(3-2y)(3+2y)$
   (d) $(2b+5)^2$
5. $0.2(x-8)(x-2)$
6. (a) $x=-7, x=-5$
   (b) $h=6, h=7$
7. (a) \$1 000 000
   (b) 2002 and 2008
   (c) 2003-2007
8. (a) $R=(1+0.1x)(60-2x)$, where x=# increases, R=revenue
   (b) \$2 per rose

Chapter 4

1. $(-5,8), x=-5, -67$, maximum
2. $y=\frac{1}{2}(x-4)^2-2$



3. reflected in x-axis, vertical compression factor 2, shifted 6 units left
4. (a) $(2,17)$
   (b) $\left(\frac{7}{2},-\frac{177}{8}\right)$
5. (a) $y=(x+5)^2-33$
   (b) $y=2\left(x+\frac{9}{4}\right)^2-\frac{81}{9}$
   (c) $y=-3\left(x-\frac{5}{6}\right)^2+\frac{109}{12}$
6. (a) 10, -2
   (b) 0.722, -0.347
7. none
8. -0.4, -3.6
9. (a) $y=-0.03x^2+12$



   (b) (i) 2.28 m
   (ii) 36.51 m
10. 1 m

Chapters 5-6

1. $\Delta PQR \sim \Delta PST$
   $y=\frac{341}{36}$
   $\Delta ACB \sim \Delta AED$
   $x=\frac{80}{3}$
2. 77°
3. $x \approx 53.8, \quad y \approx 34.6$
   $\theta \approx 29°, \quad \beta \approx 83°$
4. 10 m
5. 27 m
6. 32.8 cm, 0.25
7. 412 m
8. 64 km, S 33° W